

Variability, Brightness Temperature, Superluminal Motion, Doppler Boosting, and Related Issues

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Abstract. We review the observations of rapid flux density variations in compact radio sources, and discuss the inverse Compton limit to the maximum brightness temperature of incoherent synchrotron sources in comparison with recent VLBA observations. The apparent agreement of the theoretical brightness temperature limit due to inverse Compton cooling and the brightness temperatures observed by early VLBI observations appears to have been fortuitous. VLBA observations have greatly improved the quality of the data, but many of the early issues remain unresolved.

1. Variability Issues

Starting in the mid 1960s, it was becoming increasingly clear that many flat spectrum radio sources exhibited very rapid flux density variations (e.g., Sholomit-skii 1965; Dent 1965; Pauliny-Toth & Kellermann 1968). This presented serious problems for conventional synchrotron models as it appeared to require a huge energy content and an excessive amount of inverse Compton cooling.

1.1. The Saga of 3C 120

One of most variable sources observed in the early years was PKS 0430+05 (Day et al. 1966) also known as NRAO 182 (Pauliny-Toth et al. 1966), which is now widely, but incorrectly, known as 3C 120. The misunderstanding about the nomenclature of this source goes back to the program of flux density and variability observations made with the newly completed 140-ft radio telescope at 10, 6, and 2 cm. The observing list for these programs was created by combining the source lists which Ivan Pauliny-Toth and I had separately made from our previous work at the 300 ft (mostly norther sources) and Parkes telescopes (mostly southern sources) respectively. One source was common to our two lists, but was called NRAO 182 by Pauliny-Toth and PKS 0430+05 by myself. Thinking that such a strong centimeter source should have been included in the better known low frequency catalogs, we searched in vain for a source at the same or nearby position in the 3CR catalog (Bennett et al. 1962)

Our inspection of the original confusion limited 3C Catalog was, at best, confusing. The position (B1950) given in the NRAO VLA Calibrator Manual is $RA = 04^{\text{h}}30^{\text{m}}31^{\text{s}}.6$ and $Dec = +05^{\circ}14'59''.6$. The position of 3C 120 in the original 3C Catalog is $RA = 04^{\text{h}}29^{\text{m}}32^{\text{s}} \pm 4^{\text{s}}$ and $Dec = +01^{\circ}55' \pm 8'$ which is more than 3 degrees away in declination. However, due to noise and confusion, the positions of some sources in the 3C Catalog can be in error by one or more

full lobe shifts of the interference pattern. At +5 degrees declination, $\Delta RA = 57^s$ and $\Delta Dec = 2^\circ 13' 59''$. Applying a correction of one lobe shift in each coordinate and allowing for an additional 2 sigma error in declination ($16'$) gives $RA = 04^h 30^m 29^s \pm 4^s$ and $Dec = +05^\circ 12' \pm 8'$. Although this is in reasonable agreement with the modern VLA position, after allowing for a lobe shift in both coordinates as well as a 2 sigma shift in declination, a 3C source can be found at almost any position in the sky. So, there is probably no connection between the original 3C 120 and the variable superluminal source which has received so much attention over the past three decades. Nevertheless, just for our own bookkeeping, somewhat frivolously we called our source “3C 120” in order not to favor either one of us with the NRAO or Parkes designation.

Subsequently, we discovered remarkably rapid flux density outbursts in “3C 120” with observed increases of as much as 1 Jy per week at 2 cm (Pauliny-Toth & Kellermann 1968). We were able to fit the change in peak flux density with frequency and time with a simple expanding source model (Shklovskii 1965; van der Laan 1966; Pauliny-Toth & Kellermann 1966) with multiple outbursts each separated by about a year. We were excited by these observations, and rushed into publication, having forgotten that we had essentially *invented* the name 3C 120 for this source. Since then, numerous authors have used this nomenclature in hundreds of subsequent publications (see NED). Apparently no one ever went back to look for 3C 120 in the original 3C or 3CR catalogs. This is a good opportunity to set the record straight. There is no 3C 120!

Curiously, no radio source since, has shown the same simple behavior. Rather most sources have multiple outbursts that overlap in frequency and time so that there is no unique decomposition into individual outbursts. In spite of the early success in interpreting 3C 120 and 1934–63 (Shklovskii 1965), and other sources, it isn’t clear how to accommodate the expanding source model within currently popular shock in jet models (e.g., Marscher & Gear 1985). Unfortunately, there were no VLBI observations at that time to study how the structure changed with time. Were there really multiple expanding components? If so, were they coincident in space, or did they propagate along a jet-like path? What was the size and expansion velocity of these transient outbursts? These questions partially motivated the development of the first NRAO-Cornell VLBI system. But, by the time the MkI VLBI system became operational in 1967, the isolated outbursts in 3C 120 had ceased. In subsequent years the outbursts have been weaker, and like other quasars and AGN individual outbursts overlapped in both frequency and time.

2. The Inverse Compton Catastrophe and Superluminal Motion

As 3C 120 is relatively nearby, it did not present any particular energetic problems. Other sources, such as 3C 279 appeared to require enormous energy, with up to 10^{60} erg in the form of relativistic particles generated in less than one year (Pauliny-Toth & Kellermann 1966). Moreover, causality arguments placed limits on the size and corresponding radiation density of these transient sources. The amplitude and variability time scales of the observed outbursts appeared to be inconsistent with the calculated energy losses due to inverse Compton scattering (Hoyle, Burbidge, & Sargent 1966). It was particularly difficult to understand

the variations that we and others observed at the longer wavelengths (Pauliny-Toth & Kellermann 1966; Fanti 1979; Hunstead 1972). These issues were exacerbated with the discovery of even more rapid intraday variability (IDV) (see Jauncey et al., these proceedings, page 199). Although now both the low frequency variability and IDV are thought to arise from scattering in the ISM, the required dimensions still strain conventional theory.

The discovery of superluminal motion (Cohen et al. 1971, Whitney et al. 1971) offered a simple solution in terms of relativistic beaming (Cohen et al. 1977) following ideas suggested earlier by Rees (1966; 1967) and Woltjer (1966). However, recently, more extensive observations show little or no direct relation between variability time scales and observed velocity for individual sources; but, there remain ambiguities in the definition of the variability time scales. It is encouraging, however, that the distribution of observed velocities and variability time scales is consistent with a realistic distributions of Lorentz factors (Cohen et al., these proceedings, page 177).

3. $T_b(\text{max})$ and Inverse Compton Cooling

Kellermann & Pauliny-Toth (1969) expressed the inverse Compton cooling arguments in terms that could be directly related to observations. We showed that for any realistic magnetic field strength that the ratio of inverse Compton radiation to synchrotron radiation is simply a function of the peak brightness temperature, T_b . We pointed out that if $T_b > 10^{12}$ K, the relativistic electron population would be quickly quenched by inverse Compton cooling. Early VLBI observations appeared to indicate that for the compact flat spectrum radio sources, $T_b \sim 10^{11-12}$ K. However, with the benefit of 30 years of hindsight, both the derivation and the apparent agreement with VLBI observations were due to a series of fortuitous accidents.

Ivan and I did not intend to discuss the brightness temperature of synchrotron sources. Rather, we wanted to show from our multi-frequency flux density observations, that the flat spectra observed for many sources actually consisted of multiple peaks and valleys; and that this was due to the superposition of multiple components each of which becomes optically thick at different wavelengths.

This was in the very early days of VLBI, but multi-baseline multi frequency observations were already showing that, as expected from synchrotron theory, that components with the highest self absorption cutoff frequency, had the smallest angular diameter (Kellermann et al. 1969). Partly as a test of synchrotron radiation models, and partly to see if we could determine the magnetic field strength in the individual components, we plotted the spectral cutoff frequency, ν_m vs. S/θ^2 . As shown in Figure 1, we also plotted lines of constant brightness temperature. Using the relation between angular size, surface brightness, and magnetic field, we showed lines of constant magnetic field strength, and we noted that the observed angular size and self absorption cutoff frequencies were consistent with $B \sim 10^{-5}$ to 10^{-4} Gauss. We were surprised to see that the observed values of T_b were essentially all in the narrow range 10^{11} to 10^{12} K.

We spent several weeks trying to understand what was special about $T_b \sim 3 \times 10^{11}$ K. Finally, we realized that with a simple substitution of variables

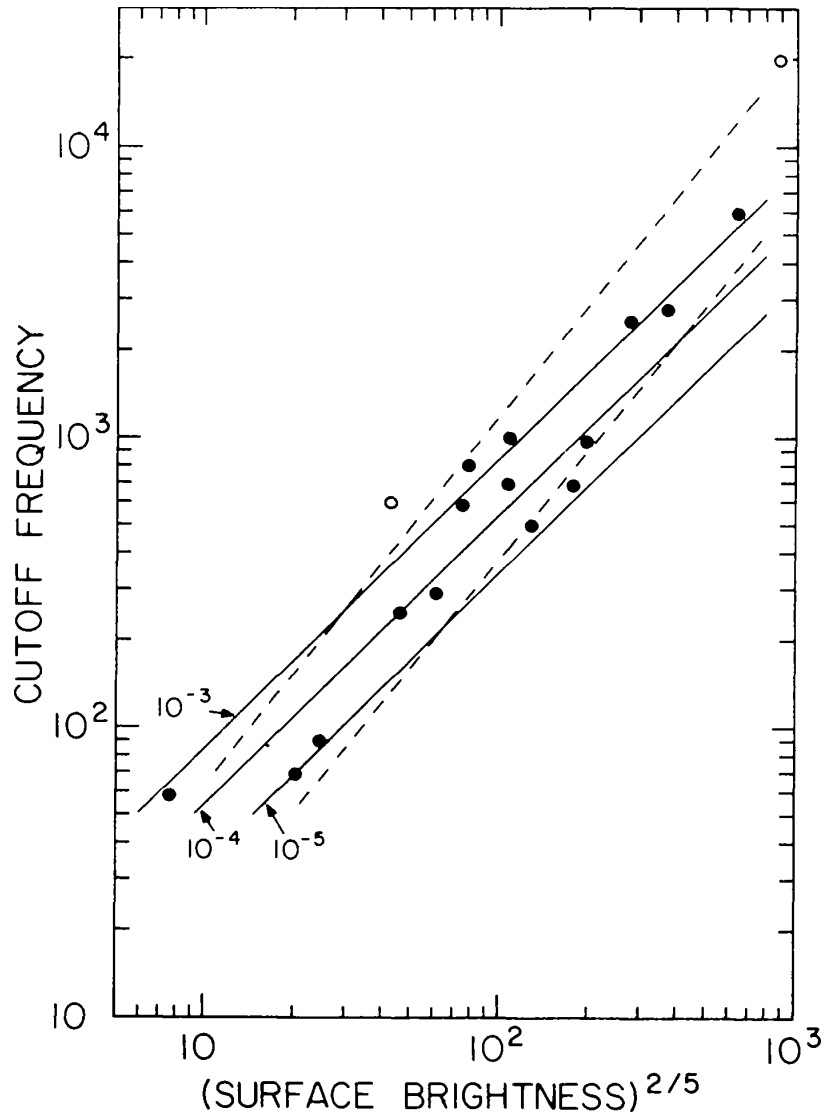


Figure 1. Spectral cutoff frequency (MHz) vs. $(\text{surface brightness})^{2/5}$ for a number of opaque sources or components of sources. The solid parallel lines show the expected dependence for different values of the magnetic field. The upper and lower dashed lines represent constant brightness temperatures of 10^{11} and 10^{12} K respectively. Taken from Kellermann & Pauliny-Toth 1969, *ApJ*, 155, L71.)

the ratio of inverse Compton scattering to synchrotron radiation, L_c/L_s can be expressed simply in terms of the observed brightness temperature. Hoyle, Burbidge, & Sargent (1966) had noted that if $L_c/L_s > 1$, then the effect of inverse Compton cooling diverges, so we realized that values of $T_b > 10^{12}$ K cannot be sustained.

With the benefit of hindsight, the agreement between our derivation of $T_b(\text{max})=10^{12}$ K and the apparent maximum observed brightness temperature

was fortuitous and was the result of the following over-simplifications and coincidences.

1) The limiting brightness temperature depends on the electron energy distribution, the upper energy limit, and the detailed geometry. We made our calculations for a uniform slab geometry as that was the only geometry for which we could handle the math, and reported that $T_b(\text{max})$ is in the range 10^{11-12} K, not 10^{12} K as is often stated in the literature. Based on more detailed calculations, Readhead (1994) estimates 3×10^{11} K as the limiting brightness temperature due to inverse Compton cooling.

2) There is no reason to assume that the VLBI observations were made at the peak of the spectrum. Below the peak, the flux density falls off rapidly as $\nu^{2.5}$. Above, the cutoff frequency, the brightness temperature also falls off rapidly as $\nu^{-2.7}$. Thus the observed brightness temperature is likely to be less than the actual peak value.

3) The early VLBI observations used in our study had only a limited number of visibility measurements which we interpreted in terms of simple circular Gaussian components. We now know, of course, that the true structure is more complex, usually with multiple separated components. Thus, the apparent sizes that we used to calculate T_b reflected the component separation rather than the size of any individual component, and so it should not be used to calculate T_b .

4) The inverse Compton limit to the maximum observed brightness temperature refers only to a source at rest. Doppler boosting increases the observed brightness temperature by the Doppler factor, δ . Most of the sources which we included in our 1969 analysis are now known to show superluminal motion, (see Zensus et al., these proceedings, page 27). These sources have significant Doppler factors, so their brightness temperature can significantly exceed the inverse Compton limit.

5) There is growing evidence that that in some sources, at least, the maximum brightness temperature is limited by free-free absorption from an intervening ionized medium rather than by synchrotron self absorption, (e.g., Vermeulen et al. 2003; Shaffer, Kellermann, & Cornwell 1999; Lister, these proceedings, page 71). However, these sources also have measured angular sizes which are consistent with self absorption, so that there appears to be a mixture of SSA and FFA taking place. It is curious however, that in these sources, the FFA and SSA opacities appear to be comparable.

6) Coherent processes, which can increase the observed brightness temperature above 10^{12} K may be important (e.g., Melrose 2002).

7) Synchrotron radiation from ultra relativistic protons can reach a peak brightness temperature which is about a factor of $(m_p/m_e)^{9/7} \sim 10^4$ higher than from electrons (Kardashev 2000). However, in this case, the required magnetic field is much larger by a factor of $(m_p/m_e)^2$ which is probably unrealistic.

8) Pauliny-Toth & Kellermann (1966) noted that for a source at rest, causality arguments indicated that the energy in relativistic electrons greatly exceeds that in the magnetic field and would be unrealistically large. The effect of Doppler boosting is to increase the apparent angular size and magnetic field strength, and so lower the relativistic particle energy requirements by $\delta^{-7/2}$. Doppler factors $\delta \sim 10$ will typically bring the particle and magnetic energies

into equilibrium. In this case the the peak brightness temperature is reduced by about a order of magnitude over the inverse Compton limit (Readhead 1994).

9) As we pointed out in our 1969 paper (see also Slysh (1992), the inverse Compton limit only applies after the source has reached equilibrium. For a limited period following the injection of a new supply of relativistic electrons, or if there is continuous reacceleration, brightness temperatures may be observed which are in excess of the inverse Compton limit. In this case, copious X-ray emission should be observed. For example first order Compton scattering will allow T_b to exceed 10^{12} K for about one year following the injection of relativistic electrons. Higher order scattering will reduce the lifetime further, but the scattering is ultimately limited by quantum effects.

10) Finally, we point out that the measured values of brightness temperature which prompted our discussion of $T_b(\text{max})$ and inverse Compton cooling was merely the natural consequence of the size of the Earth and the limited range of flux density observed, and had nothing to do with radio source physics. This is because the resolution of any interferometer is given by $\theta = \lambda/D$ where λ is the wavelength of observation, and D is the baseline length. For a radio source of flux density S and angular size, θ , the brightness temperature $T_b = 2k\lambda^2 S/\theta^2$. Thus $T_b \propto SD^2$, and is independent of angular size and observing wavelength. For $1 < S_{\text{Jy}} < 10$, and $D \sim 5000$ km, $T_b \sim 10^{11-12}$ K.

4. Current Issues

More than three decades have passed since Ivan and I considered the question of a limiting brightness temperature. In many ways we are still asking the same questions about the same issues. However, due to tremendous advances in both the observational data and theoretical models, our questions are at a much higher level than before. Although the observations sometimes strain conventional synchrotron models and relativistic boosting, there does not appear to be any show stoppers. However, the number of free parameters needed to accommodate the observations has become uncomfortably large, and it has proven difficult to confirm by observation the predicted effects of relativistic motion. The early models make simple predictions. But, today, consideration of jet instead of ballistic flows, possible differences between the bulk velocity and the pattern velocity, combined with a distribution of intrinsic Lorentz factors has removed the predictability, and thus the simplicity, if not the attractiveness of these models.

It is perhaps sobering to recall that the concept of relativistic beaming was first introduced in the mid 1960's to explain the rapid variability that was observed in quasars, and which appeared to be dramatically confirmed by the discovery of superluminal motion in 1971. But, predictions of the distribution of observed velocity distribution can only be reconciled with observations by introducing modifications to the simple theory which involves new free parameters. Meanwhile simple non-Doppler boosting models such as suggested by Ekers and Liang (1989) are surprisingly consistent with the observed velocity distribution. The main evidence for Doppler boosting remains, as in the 1960's, the rapid flux density variations, not superluminal motion.

As discussed by Cohen et al. (these proceedings, page 177) application of the standard causality argument combined with the measured velocities can be used to estimate the intrinsic brightness temperature and Doppler factor. Although there is no simple direct relationship between the brightness temperatures calculated from variability time scales and measured velocities, the observed values are consistent with a reasonable distribution of Lorentz factors. However ambiguities in defining the variability time scales, in relating the variable component to the moving component, and the apparent differences between the bulk velocity and pattern velocity may preclude any meaningful quantitative conclusions.

Modern VLBA (Kellermann et al; 1998, Zensus et al. 2002; Kovalev, these proceedings, page 65) and VSOP space VLBI (Tingay et al. 2001) observations suggest maximum observed brightness temperatures in the range 10^{11-13} , much as we measured in the first years of VLBI. Although VSOP uses longer baselines, fringe visibilities are more accurately determined on the VLBA so the corresponding size limits for unresolved components are comparable for the two instruments. The Russian *RadioAstron* space VLBI mission which is scheduled for launch in 2006 will give more than an order of magnitude increase in baseline length. This will directly measure brightness temperatures up to 10^{15} K and perhaps allow better comparison with calculated Doppler factors.

The maximum relativistic boosting occurs when the the motion is oriented directly along the line of sight. In this case, $T_b(\text{obs}) \sim 2\gamma T_b(\text{int})$ where γ is the Lorentz factor. The observed range of T_b is consistent with the observed range of apparent velocity and intrinsic values of $T_b \sim 10^{10-11}$ K, or close to both the inverse Compton limit and the equilibrium value. However, perhaps surprisingly, there is no simple one-to-one relation between the observed brightness temperature and measured superluminal motions.

Using the same kind of causality arguments used in the 1960s to “predict” superluminal motion, IDV observations suggest brightness temperatures as much as $\sim 10^{21}$ K they are intrinsic and $T_b \sim 10^{15}$ K if the variations are due to interstellar scintillations as seems likely, (Jauncey et al., these proceedings, page 199). It may be hard to reconcile these numbers with reasonable Doppler factors.

In principle, the intrinsic brightness temperature and Doppler factor can be determined from the ratio of observed inverse Compton X-ray flux density to the radio flux density from synchrotron radiation, (e.g., Unwin et al. 1997). However, there are three problems in implementing this technique. 1) It is difficult to get contemporaneous radio and X-ray light curves. 2) We don’t know whether the X-rays are coming from the core or from somewhere along the jet. 3) There may be an unknown additional (non-Compton), amount of X-ray emission for example from the hot ionized torus surrounding the central black hole.

The concept of equipartition in extragalactic radio sources was first introduced by Burbidge (1959) to minimize the total energy content of extended lobe radio sources which typically have ages of 10^{8-9} years. We do not know if it is appropriate to assume that equipartition conditions apply in the short lived compact cores and jets. Indeed, the presence of bulk relativistic motion in the parsec scale jets suggests that the plasma may not be contained by the magnetic field so that there may be an excess of particle energy over magnetic energy. Comparison of Doppler factors deduced from variability time scales and measured proper motions suggests that the intrinsic brightness temperatures may

be closer to the equipartition value than to the inverse Compton limit (Cohen et al., these proceedings, page 177), but the difference between the two is small and perhaps not meaningful.

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