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Towards a Unified Source-Propagation Model of Cosmic Rays

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Abstract. It is well known that the cosmic ray energy spectrum is multifractal with the analysis of cosmic ray fluxes as a function of energy revealing a first "knee" slightly below 10^{16} eV, a second knee slightly below 10^{18} eV and an "ankle" close to 10^{19}eV. The behaviour of the highest energy cosmic rays around and above the ankle is still a mystery and precludes the development of a unified source-propagation model of cosmic rays from their source origin to Earth. A variety of acceleration and propagation mechanisms have been proposed to explain different parts of the spectrum the most famous of course being Fermi acceleration in magnetised turbulent plasmas (Fermi (1949)). Many others have been proposd for energies at and below the first knee (Peters & Cimento (1961); Lagage & Cesarsky (1983); Drury et al. (1984); Wdowczyk & Wolfendale (1984); Ptuskin et al. (1993); Dova et al. (0000); Horandel et al. (2002); Axford (1991)) as well as at higher energies between the first knee and the ankle (Nagano & Watson (2000); Bhattacharjee & Sigl (2000); Malkov & Drury (2001)). The recent fit of most of the cosmic ray spectrum up to the ankle using non-extensive statistical mechanics (NESM) (Tsallis et al. (2003)) provides what may be the strongest evidence for a source-propagation system deviating significantly from Boltmann statistics. As Tsallis has shown (Tsallis et al. (2003)), the knees appear as crossovers between two fractal-like thermal regimes. In this work, we have developed a generalisation of the second order NESM model (Tsallis et al. (2003)) to higher orders and we have fit the complete spectrum including the ankle with third order NESM. We find that, towards the GDZ limit, a new mechanism comes into play. Surprisingly it also presents as a modulation akin to that in our own local neighbourhood of cosmic rays emitted by the sun. We propose that this is due to modulation at the source and is possibly due to processes in the shell of the originating supernova. We report that the entire spectrum, spanning cosmic rays of local solar origin and those eminating from galactic and extra-galactic sources can be explained using a new diagnostic - the gradient of the log-log plot. This diagnostic reveals the known Boltmann statistics in the solar-terrestrial neighbourhood but at the highest energies - presumably at the cosmic ray source, with clearly separated fractal scales in between. We interpret this as modulation at the source followed by Fermi acceleration facilitated by galactic and extra-galactic magnetic fields with a final modulation in the solar-terrestrial neighbourhood. We conclude that the gradient of multifractal curves appears to be an excellent detector of fractality.

1. Theory

1.1. Heuristic Approach

The Boltzmann-Gibbs (B-G) equilibrium distribution

$$p_i = e^{-\beta E_i}/Z \tag{1}$$

with $\beta \equiv 1/kT \geq 0$, state energy E_i and partition function $Z \equiv \sum_j e^{-\beta E_j}$ can be obtained (Tsallis et al. (1998)) (ignoring the trivial normalisation factor $1/Z$) from the solution of the linear ordinary differential equation (ODE),

$$dp_i/dE_i = -\beta p_i. \tag{2}$$

For thermodynamically anomalous systems with generalised entropic form $S_q = k(1 - \sum_i p_i^q)/(q-1)$ then, optimisation under appropriate constraints yields instead the NESM power-law distribution,

$$p_i \propto [1-(1-q)\beta_q E_i]^{\frac{1}{1-q}} \equiv e_q^{-\beta_q E_i} \tag{3}$$

(definition (Tsallis et al. (1998))) that recovers the B-G statistical weights for $q = 1$ by setting $\beta_q \equiv \beta$. As above, this distribution can also be obtained from the solution of the nonlinear ODE,

$$dp_i/dE_i = -\beta_q p_i^q \tag{4}$$

with $\beta_q \geq 0;\ q_1 \geq 1$. In Tsallis et al. (2003), this heuristic approach was taken further to create a crossover from B-G ($q = 1$) to NESM ($q \neq 1$) for $q > 1$ and $\beta << \beta_q$ with the ODE,

$$dp_i/dE_i = -\beta p_i - (\beta_q - \beta)p_i^q. \tag{5}$$

The subsquent generalisation,

$$dp_i/dE_i = -\beta_{q'} p_i^{q'} - (\beta_q - \beta_{q'})p_i^q, \tag{6}$$

then allowed for the production of the additional crossover at the first knee allowing Tsallis (2003) to obtain the excellent fit of the spectrum up to the second knee shown in Figure 1.

1.2. Generalised multifractal spectra

Generalising the heuristic approach above so that at lowest energies, B-G statistics dominates and as we go to higher energies, we allow for a transition from B-G to NESM statistics progressing through a series of $n-1$ cross-overs from one anomolous scale-free region to another as follows,

$$dp_i/dE_i = -\beta_n p_i^{q_n} - \sum_{j=0}^{n-1}\left[(\beta_j - \beta_{j+1})p_i^{q_j}\right] \tag{7}$$

then, for the case $n = 3$ we can extend the Tsallis source-propagation model of the cosmic ray energy spectrum to include also a third crossover at the ankle whereby,

$$dp_i/dE_i = -(\beta - \beta_1)p_i - (\beta_1 - \beta_2)p_i^{q_1} - (\beta_2 - \beta_3)p_i^{q_2} - \beta_3 p_i^{q_3}. \tag{8}$$

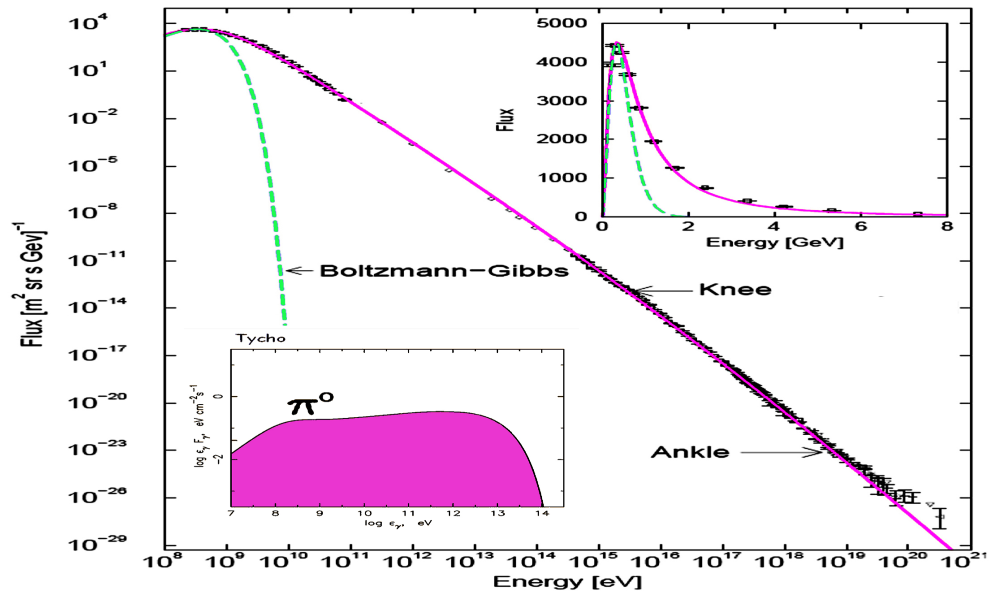

Figure 1.: The excellent single-theory fit to the cosmic ray flux-energy spectrum provided by NESM (Tsallis et al 2003). The energy spectrum of π^0 particles in the direction of the Tycho SNR (inset) shows that the pion spectrum coincides with the NESM (galactic) propagation region up to the first knee.

1.3. Solution of ODEs

The exact solutions of ODEs like those above are in general given by $p_i \propto f(E_i)$ where $f^{-1}(x)$ is an explicit monotonic function of x involving hypergeometric functions. The first step involves writing the integral equation for the general ODE above,

$$E_i = \int_{p_i}^{1} \frac{dx}{\beta_n x^{q_n} + \sum_{j=0}^{n-1} \left[(\beta_j - \beta_{j+1}) x^{q_j} \right.}. \tag{9}$$

This can be solved using Gauss hypergeometric series,

$${}_2F_1\Big[(A,B);C;z\Big] = \sum_{n=0}^{\infty} \frac{(A)_n (B)_n}{(C)_n} \frac{z^n}{n!} \tag{10}$$

expressed in terms of the rising factorials or Pockhammer symbols: $(A)_n$, $(B)_n$ and $(C)_n$ such that for example $(A)_n = A(A+1)(A+2)...(A+n-1)$, $(A)_0 = 1$. Expressed in this series form, $E = f(p_i)$ can be written compactly.

1.4. Relating the solutions p_i to the cosmic ray flux

The flux $\Phi(E)$ can be obtained straightforwardly from $p_i \propto f(E_i)$ by calculating the density of states $\omega(E)$. The density of states of an ideal gas in three dimensions is given by $\omega(E) \propto E^2$, hence $\Phi(E) = AE^2 f(E)$, where A is a normalising factor (and where red shift effects have been neglected). With this expression we fit the observational data up to the highest energies as shown in Figure 2.

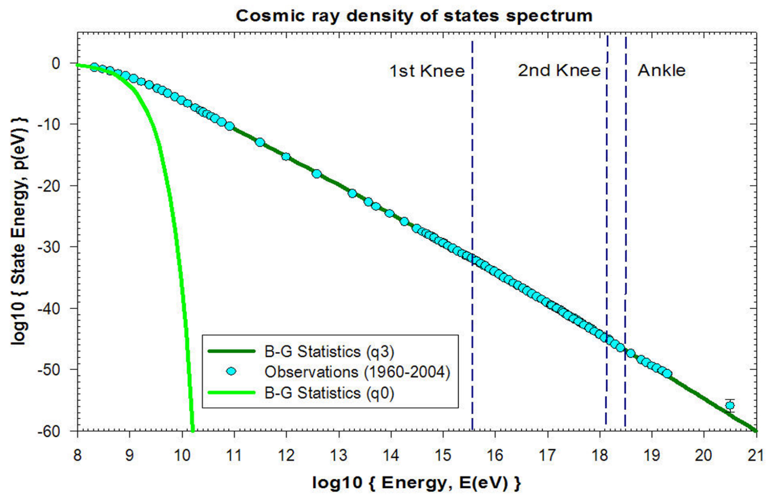

Figure 2.: The improved fit to the cosmic ray spectrum at the highest energies in the ankle region provided by the generalisation of the Tsallis theory.

1.5. Gradient of the logarithmic energy spectrum

The cosmic ray energy spectrum is usually plotted on logarithmic axes such that scale-free regions correspond to linear portions of the spectrum. It has already been shown that there are at least 2 such anomalous regions split by a cross-over at the knee. In fact, a plot of ***the gradient of the logarithmic cosmic ray energy spectrum is even more revealing***. In order to construct such a plot, it is necessary to calculate the gradient m,

$$m = \frac{d(logp_i)dp_i}{d(logE)dE} = \frac{dp_i}{dE}\Big(\frac{E}{p_i}\Big). \tag{11}$$

Horizontal regions of a plot of m versus $logE$ then reveal scale-free regions. Note that dp_i/dE is just the generating ODE so that for the case $dp_i/dE_i = -\beta_{q'}p_i^{q'} - (\beta_q - \beta_{q'})p_i^q$ then,

$$\begin{aligned} m &= \Big[-\beta_{q'}p_i^{q'} - (\beta_q - \beta_{q'})p_i^q\Big]\Big(\frac{E}{p_i}\Big) \\ &\equiv -E\Big[\beta_{q'}p_i^{q'-1} + (\beta_q - \beta_{q'})p_i^{q-1}\Big] \end{aligned} \tag{12}$$

and so forth. Figure 3 shows the result. The simple logarithmic plot is deceptive to the eye. The log-gradient plot on the other hand shows us scale-free regions more clearly. The fit given by q_3 to the highest energy cosmic rays is almost perfectly Bolzmann-Gibbsian suggesting that the whole cosmic ray spectrum should be divided into 4 district physical domains as shown in Figure 4.

2. Discussion

The generalisation of the NESM heuristic equation to higher orders has provided an excellent fit over the whole cosmic ray spectrum with a single differential equation whose

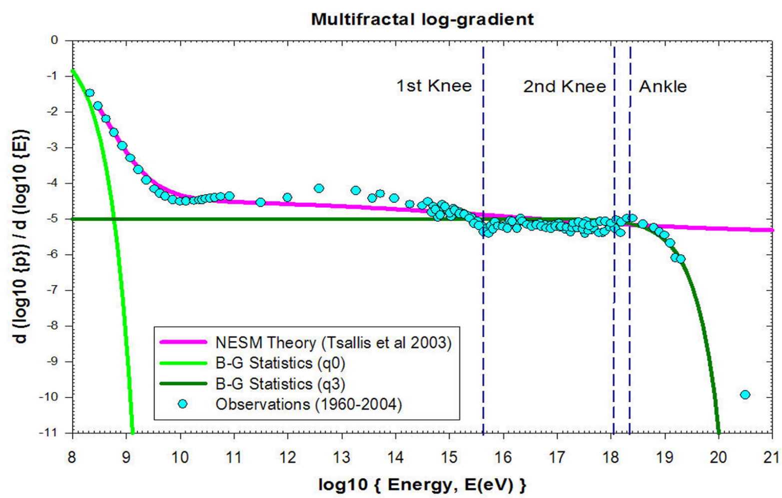

Figure 3.: The multifractal log-gradient model indicating clearly the improvement over the earlier second order Tsallis model. At the highest energies, the data just below the GDZ cutoff is almost perfectly Boltzmann-Gibbsian suggesting a strong association with cosmic ray source physics.

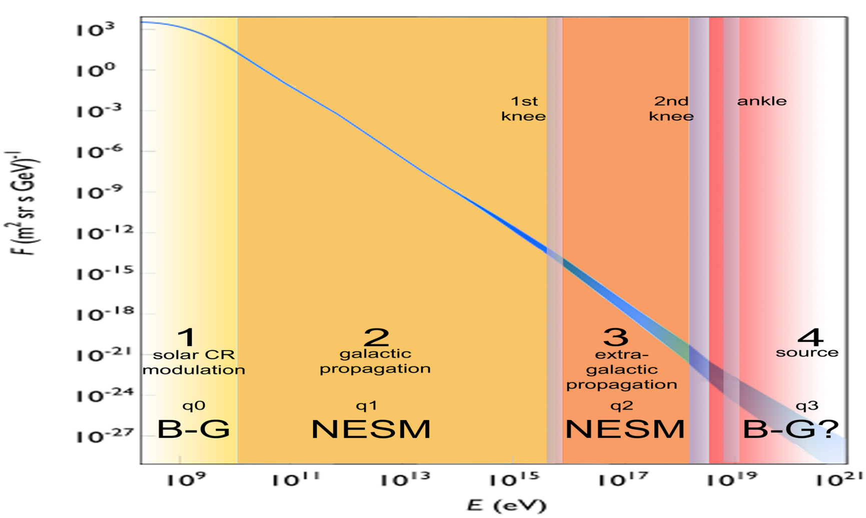

Figure 4.: Regions of interest in the cosmic ray energy spectrum demarcated by this work into solar and high energy cosmic ray source regions and NESM propagation regions: 1=solar cosmic ray source and modulation region (q_0), 2=anomalous NESM (galactic) propagation region (q_1), 3=anomalous NESM (extragalactic) propagation region (q_2), 4=cosmic ray source region (q_3).

solution, although mathematically difficult to obtain, is expressable in terms of simple hypergeometric series. The solution incorporates both the (Boltzmann-Gibbsian) physics of high energy source cosmic rays, their mediation through various regions and their modulation by solar cosmic rays in our own space neighbourhood suggesting a potentially new interpretation of the cosmic ray energy spectrum.

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