# May Wide Triple Star Systems Be Flat? 

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#### Abstract

We studied the possibility of flat solutions for triple systems. For the analysis we chose the systems of type I: a close pair with a distant companion, with known orbits where the satellite shows significant relative motion. We calculated two variants of the orbit for the wide pairs - with the condition of coplanarity of orbits of close and wide pair, and without it. If the sum of squared residuals for orbit determined with the condition is not increased, it may be assumed that the system is flat. Among reviewed fifteen triple systems, the following ones may be flat: WDS 20474+3629, 00321+6715, 01198-0031, $02291+6724, ~ 05239-0052, ~ 06003-3102, ~ 08122+1739,10370-0850$, $17066+0039$. Other systems are not able to have the coplanar orbits of wide and close pair: WDS $8592+4803,01350-2955,02022+3643,04400+5328$ and $08468+0625$. Three of them have a retrograde motion.


## 1. Introduction

The most frequent triple star systems are hierarchical (type I): a close pair and distant companion with an orbital period of hundreds or thousands of years. The planes of the orbit of close and wide pairs, calculated using observations, do not coincide in numerous wide visual multiple systems. Information about relative orientation of the orbits of wide and close pairs in triple systems is important for study of formation, evolution and stability of multiple stars. The orbits of close pairs with small rotation periods are determined confidently, while the determination of distant companion orbit is difficult due to insignificant apparent relative motion during the observation period. Also the observation errors may lead to large uncertainties of orbital parameters and, consequently, to the uncertain orientation of the wide pair orbital plane. As the inclination and longitude of the ascending node of the orbit may be correlated, and distribution of errors is unknown, it is not possible to correctly estimate the probability that the two orbital planes are really the same.

The purpose of this work is to find for the selected triple systems the flat solutions that are in good agreement with observations. For the triple systems we calculated the orbit of the close pair at first step. Then the two variants of the distant orbit of the satellite are determined for wide pairs. Both variants were calculated by Thiele-Innes method with further determination of differential corrections. The first orbit was determined with condition of minimum the sum of squared residuals between the observed and the calculated position of the distant companion. The second variant of distant companion orbit was determined with the condition that the plane of the orbit coincides with the plane of the orbit of close pair ( $i$ and $\Omega$ are equal to the values already known for close pair).

### 1.1. Selected Multiple Stars

To investigate the coincidence of the orbital planes we selected the fifteen multiple systems (two systems are quadruples and others are the triples) with known orbits (Hartkopf 2001) ${ }^{1}$. At selection we took into account the relative motion of the pair in close and distant component. The relative motions in the close pairs must be more than $75 \%$ of the apparent ellipse during the observation period. The observed arcs of wide pairs are greater than $15 \%$ of the full ellipse. Selected systems are presented in the Table 1.

Table 1. List of the multiple systems. $1 / \mathrm{L}$ — ratio of observed arc to full ellipse.

| n | WDS | Close pair | l/L | Wide pair | l/L |
| :--- | :--- | :--- | :--- | :--- | :---: |
| 1 | $22288-0001$ | $\mathrm{Aa}, \mathrm{Ab}$ | 1.43 | AB | 0.54 |
| 2 | $08592+4803$ | BC | 2.87 | $\mathrm{~A}, \mathrm{BC}$ | 0.18 |
| 3 | $20474+3629$ | $\mathrm{Aa}, \mathrm{Ab}$ | 1.11 | AB | 0.37 |
| 4 | $00321+6715$ | $\mathrm{Aa}, \mathrm{Ab}$ | 1.54 | AB | 0.31 |
| 5 | $01198-0031$ | $\mathrm{~A}, \mathrm{BC}$ | 2.13 | BC | 0.20 |
| 6 | $01350-2955$ | AB | 20.85 | $\mathrm{AB}, \mathrm{C}$ | 1.06 |
| 7 | $02022+3643$ | AB | 8.28 | $\mathrm{AB}, \mathrm{C}$ | 0.34 |
| 8 | $02291+6724$ | $\mathrm{Aa}, \mathrm{Ab}$ | 0.75 | AB | 0.26 |
| 9 | $04400+5328$ | AB | 4.28 | $\mathrm{AB}, \mathrm{C}$ | 0.31 |
| 10 | $05239-0052$ | $\mathrm{~A}, \mathrm{BC}$ | 2.44 | BC | 0.15 |
| 11 | $06003-3102$ | AB | 1.60 | AC | 0.86 |
| 12 | $08122+1739$ | AB | 3.15 | $\mathrm{AB}, \mathrm{C}$ | 1.62 |
| 13 | $08468+0625$ | AB | 8.25 | $\mathrm{AB}, \mathrm{C}$ | 0.31 |
| 14 | $10370-0850$ | $\mathrm{Aa}, \mathrm{Ab}$ | 1.34 | AB | 0.46 |
| 15 | $17066+0039$ | AB | 0.90 | $\mathrm{Ba}, \mathrm{Bb}$ | 0.35 |

### 1.2. Orbit Determination

The orbits of close and wide pairs were recalculated on the base of relative positions from WDS catalog Mason (2001) using Thiele-Innes method. At first, the Kepler's constant C was determined from observational sets. Position angles $\theta$ were corrected for precession and were reduced to equinox 2000.0. By choosing the some selected points on observed arc the annual mean motion $\mu$ and values for eccentric anomaly $E$ for corresponding moments were calculated. It allows to obtain the dynamical elements $(e, P, T)$. The Thiele-Innes constants $A, B, F, G$ and its errors were calculated by least squares method from equations system:

$$
x_{i}=a\left(A X_{i}+F Y_{i}\right) \quad y_{i}=a\left(B X_{i}+G Y_{i}\right)
$$

where $i=1 . . N, N$ - number of observed positions,

$$
\begin{array}{cc}
X_{i}=\cos E_{i}-e & Y_{i}=\sqrt{1-e^{2}} \sin E_{i} \\
x_{i}=\rho_{i} \cos \theta_{i} & y_{i}=\rho_{i} \sin \theta_{i}
\end{array}
$$

[^0]The orientation elements $i, \omega, \Omega$ and semi-axis major $a$ were calculated from ThieleInnes constants $A, B, F, G$. Improving of the orbit was carried out by calculating the corrections to the orbital elements $e, P, T$. The corrections were determined by solving the equations system by the least squares method:

$$
\sum_{j} \frac{\partial \rho_{c}}{\partial p_{j}} \Delta p_{j}=\rho_{c}-\rho_{o} \quad \sum_{j} \frac{\partial \theta_{c}}{\partial p_{j}} \Delta p_{j}=\theta_{c}-\theta_{o}
$$

where $p_{j}$ - one of the elements $(e, P, T) ; \Delta p_{j}$ - differential correction to the element $p_{j} ; \rho_{o}, \theta_{o}$ - observed positions; $\rho_{c}, \theta_{c}$ - ephemeris.

The revised orbits were compared with the previous results of other authors ${ }^{2}$. For the five pairs the orbits were significantly improved and they fit to observations better then orbits, obtained earlier. Fig. 1 shows a comparison of the rms residuals for revised orbits of wide pairs and the orbits determined previously by orhet authors.


Figure 1. RMS differences of the ephemeris and observations for wide pairs.

### 1.3. Possibility of Coplanar Orbit

To decide as far as one or the other variant is better satisfies the observations we considered the root mean square residuals and the model series. For each case of the orbit we calculated the rms residuals using the formula:

$$
\sigma_{1}=\sqrt{\frac{1}{N} \sum_{i}^{N}\left(x_{o}-x_{c}\right)_{i}^{2}+\left(y_{o}-y_{c}\right)_{i}^{2}}
$$

[^1]where $x_{o}, y_{o}$ - observed relative positions; $x_{c}, y_{c}$ - ephemeris; N - number of observed positions.

Model series were built on the basis the ephemeris with random errors, that distributed according to a normal distribution with fixed value of standard deviation as $\sigma_{1}$. The 200 model series were built on this principle, for each pair. For each model set we defined orbital parameters, ephemeris, rms differences of the observed positions and ephemeris and as a result we obtained the array of values $\sigma_{1 j},(j=1.200)$ and intervals of acceptable values of $\sigma_{1}$ (extreme values were excluded). The $90 \%$ of obtained $\sigma_{1 j}$ are in these ranges. Thus it was obtained a criterion of possibility of coplanar orbit with a corresponding value of $\sigma_{1}$ : if $\sigma_{1}$ is in this range, the orbit may be considered as a possible.

Table 2 shows the revised orbital solutions for close pairs and two variants of orbit for wide pairs. The orbit parameters obtained by other authors earlier are given for comparison. Table 2 is available online at http://puldb.ru/triples/ and contains the following data: WDS name; identifier of pair within the system; M - marker of orbit possibility ( $N$ - coplanar orbit does not agree to the observations, $Y$ - orbit is possible); $\phi$ - angle between the planes of the orbits of close and wide pairs; $\sigma_{1}$ RMS residuals O-C; orbital parameters and comments.

Table 2. Fragment of data for WDS 20474+3629. The parameters of revised orbit and orbit solutions by other authors for comparison.

| Pair | M | $\phi^{\circ}$ | $\sigma_{1}^{\prime \prime}$ | $\mathrm{P}, \mathrm{yr}$ | $a^{\prime \prime}$ | $i^{\circ}$ | $\Omega^{\circ}$ | $\mathrm{T}, \mathrm{yr}$ | e | $\omega^{\circ}$ | Comm. |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Aa}, \mathrm{Ab}$ |  |  | 0.004 | 11.7 | 0.047 | 140.08 | 136.5 | 1994.2 | 0.509 | 265.4 |  |
| Aa,Ab |  | 0.004 | 11.6 | 0.048 | 135.00 | 150.0 | 1982.2 | 0.524 | 272.0 | Bag1992 |  |
| AB |  | 11 | 0.059 | 391.3 | 0.785 | 131.44 | 145.5 | 2175.1 | 0.461 | 297.3 |  |
| AB | Y | 0 | 0.059 | 486.6 | 0.795 | 140.08 | 136.5 | 1789.4 | 0.376 | 312.7 | coplanar |
| AB |  | 8.2 | 0.061 | 391.3 | 0.777 | 133.80 | 138.6 | 1795.0 | 0.450 | 298.4 | Rab1948b |

### 1.4. Conclusions

Nine of considered multiple systems may have the coplanar orbits of close pair and distant companion: WDS 20474+3629, 00321+6715, 01198-0031, 02291+6724, 052390052, 06003-3102, $08122+1739,08122+1739,10370-0850$ and $17066+0039$. Other systems can not be flat: WDS $8592+4803,01350-2955,02022+3643,04400+5328$, $08468+0625$. Three of them have a retrograde motion.
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## References

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[^0]:    ${ }^{1}$ See also http://ad.usno.navy.mil/wds/orb6.html

[^1]:    ${ }^{2}$ See the full reference list at http://puldb.ru/triples/ReadMe.txt

