

21st Century Supermoon Estimation in R

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Abstract. Supermoon estimation is prediction of the timing and distance of a perigee full or new moon. Some existing relevant tools can be opaque (lacking implementation details), functionally limited (giving a subset of the required information) or give mutually inconsistent results. A new supermoon estimator has been implemented to address drawbacks in, and to help benchmark, such existing tools. Specifically, the case of the 21st century supermoons is handled. Implementation in R is discussed, including time system transformations, numerical precision, general optimisations (e.g., Horner's scheme) and first-class function leveraging.

1. Introduction

There is a need for a supermoon estimator that is transparent, comprehensive (provides lunar apsis and phase timings, distance) and rigorous (numerically precise, quantifies uncertainty). Existing community tools deliver some, but not all of these features. An estimation system was built to remedy this, in support of scientific supermoon-related studies.

2. Method

2.1. Meeusian Approximation

The supermoon estimation system was based on Meeusian approximation. That is, the astronomical algorithms of Jean Meeus which approximate complex solutions to lunar motion, for more pragmatic implementation (Meeus 1998). The algorithms used fall into four categories: apsis, phase and distance estimation, and supporting time system transformations.

2.1.1. Apsis Estimation

The first step in supermoon estimation was tracking the anomalistic month. The system estimates the time of lunar apogee and perigee; the time of greatest and least Earth-Moon distance respectively.

Drawing on ELP-2000/82 theory (theoretically and practically, via Fortran shared objects from Chapront-Touzé & Chapront 1983), the mean times of orbital extrema were estimated via a quartic polynomial, then refined via a set of apsis-specific periodic corrections. These were a function of lunar argument of latitude, lunar mean elongation, and solar mean anomaly.

2.1.2. Phase Estimation

Tracking the synodic month was next. The system can estimate the time of four lunar phases; full, new, first quarter and last quarter moons. As for apsides, a mean time was estimated, then refined by a complex set of periodic corrections. Phase estimation incorporated ELP-2000/82, but additionally leveraged VSOP87 solar theory (Bretagnon & Francou 1988).

2.1.3. Distance Estimation

Supermoon timings were supplemented with Earth-Moon distances. Estimates were again calculated from a mean, then corrected by periodic terms that were a function of orbital elements of the Earth, Moon and Sun. Notably, distance estimation used 46 periodic terms, from a modified form of ELP-2000/82 theory, not the hundreds of terms otherwise needed (Meeus 1998).

2.1.4. Time System Transformations

The system was designed to provide dates and times to users as Coordinated Universal Time (UTC). The Meeusian approximations however required multiple time system conversions:

- dynamical time $\leftrightarrow$ Julian ephemeris day,
- dynamical time $\leftrightarrow$ UTC, and
- Universal Time 1 $\leftrightarrow$ UTC,

Allowance was made for leap years. Additionally, leap seconds arising from the irregular slowing of the Earth's rotation rate (Sidorenkov 2005) were addressed with the polynomial predictions of Espenak & Meeus (2006), though using an exact decimal year rather than a step function approximation.

2.2. R

The supermoon estimation system was implemented in R 4.0.3 with the RStudio Integrated Development Environment 1.3.1093. R has a significant set of statistical, analysis and other packages (16,455 as of late Oct 2020) in the Comprehensive R Archive Network (<https://cran.r-project.org>). This offers potential for supplementary user analysis of the lunar data estimated with the system.

2.3. Numerical Precision and Uncertainties

Several measures were adopted to maintain the numerical precision of calculations. To reduce accumulated rounding errors, multi-term summations were generally rearranged such that they were summed in order of smallest to largest magnitude (as per, e.g. Mak 2003). Additionally, Horner's scheme (e.g. Pan 1966) was widely implemented, not only further reducing scope for rounding errors, but improving system speed. Finally, calculations were generally implemented not with R's default double precision (53 bit, van der Loo & de Jonge 2018), but with 128 bit precision via the **Rmpfr** package. Using high precision refines, e.g., time system transformations (by ≈ 1 s).

Many supermoon estimation functions returned values with associated uncertainties. R's support for functions as first-class citizens was leveraged to propagate these uncertainties properly through the system. A utility function was developed to invoke arbitrary functions with a value and associated uncertainty, and then process any results, assuming the invoked functions themselves returned additional uncertainties.

3. Results

System functions were validated against a series of expected results in Meeus (1998). Indeed, this validation revealed an unpublished error in the reference work, confirmed by the author (J. Meeus, personal communication, 18 October 2020): page 357, example 50.a, the sum of terms in Table 50.A (apogee) should be -0.4648 day.

Lunar distance estimates were compared with grey literature Meeusian sources, using test dates in 2001, 2050 and 2100. For <http://astropixels.com> and <http://www.fourmilab.com> data, all distances were consistent within the estimated uncertainties. Validation and benchmarking code has been left in the system for reference.

Testing dates in 2001, 2050 and 2100 against JPL's HORIZONS Web Interface¹ (a non-Meeusian source) yielded average differences for full moon time ≈ 21 min, perigee time ≈ 7 min and perigee lunar distance ≈ 2 km.

Figure 1, generated with R's `ggplot2`, shows example results; variation in lunar distance over 2019 and 2020. A list of perigean supermoons for 2001–2100, for an apsis-full moon separation of ≤ 12 h, is attached in the system documentation.

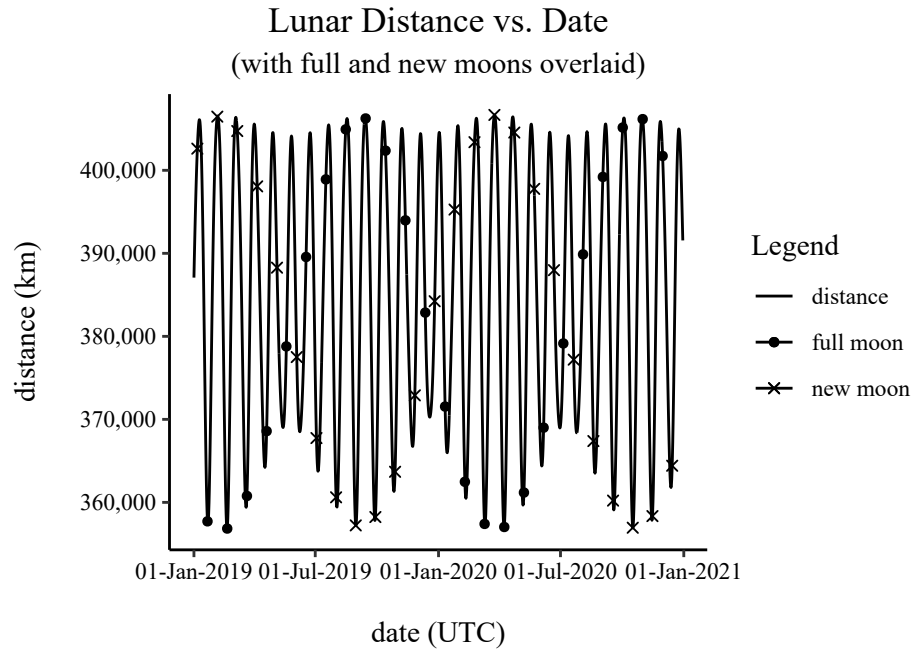


Figure 1. Lunar distance vs. date, over 2019 and 2020. A supermoon occurs when a full or new moon coincides with minimum lunar distance.

4. Discussion

A supermoon estimation tool was developed that provides timings of perigean supermoons, and thus of the Earth's maximum lunar illuminance. The system combines transparency, functional breadth and rigour. Validation and benchmarking indicate that the system is without gross logical or implementation flaws.

All lunar distance uncertainties are dynamically calculated. Apsis and phase uncertainties are however fixed approximations, and have only been confirmed as valid until the early 2020s (Meeus 1998). Refinement of these uncertainties is thus a direction of future research. For now, system source code has been uploaded to GitHub (<https://github.com/tstenborg/Supermoon>).

References

Bretagnon, P., & Francou, G. 1988, A&A, 202, 309

¹<http://ssd.jpl.nasa.gov/horizons.cgi>

- Chapront-Touzé, M., & Chapront, J. 1983, *A&A*, 124, 50
- Espenak, F., & Meeus, J. 2006, *Five Millennium Canon of Solar Eclipses: –1999 to +3000 (2000 BCE to 3000 CE)* (Hanover, MD: NASA)
- Mak, R. 2003, *Java Number Cruncher: The Java Programmer's Guide to Numerical Computing* (Upper Saddle River, NJ: Prentice Hall PTR)
- Meeus, J. 1998, *Astronomical algorithms* (Richmond, VA: Willmann-Bell), 2nd ed.
- Pan, V. Y. 1966, *Russian Mathematical Surveys*, 21, 105
- Sidorenkov, N. S. 2005, *Astronomical and Astrophysical Transactions*, 24, 425
- van der Loo, M., & de Jonge, E. 2018, *Statistical Data Cleaning with Applications in R* (Hoboken, NJ: Wiley)